

TCS 455: Quiz 1 Solution

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Semester/Year: 2/2009
Course Title: Mobile Communications
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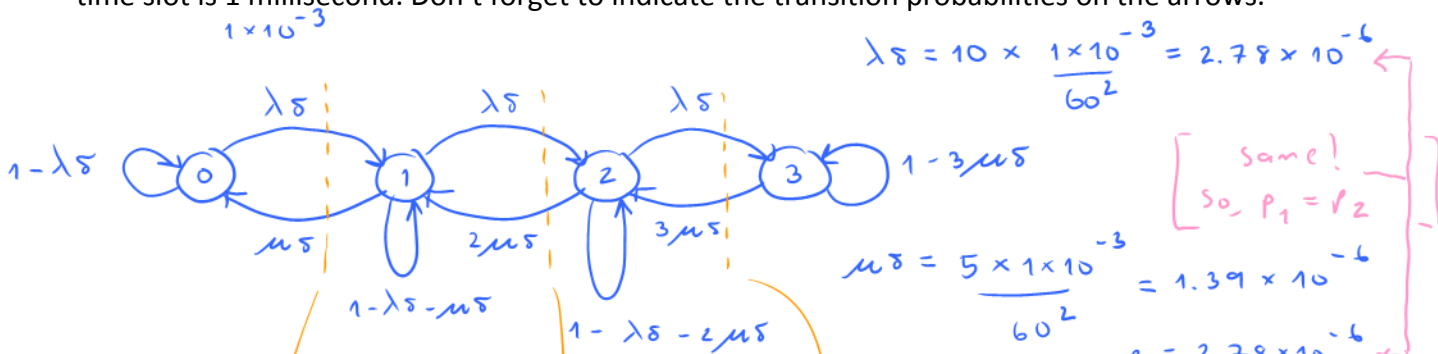
Instructions

1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group. Late submission will not be accepted.
3. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
4. **Do not panic.**

Consider a system which has 3 channels. We would like to find the blocking probability via the Markov chain method. Assume that the total call request rate is 10 calls per hour and the average call duration is 12 mins.

$$\frac{1}{\mu} = \frac{12}{60} \text{ hr} = \frac{1}{5} \text{ hr.} \quad \frac{\lambda}{\mu} = 2$$

1. **Draw the Markov chain** via discrete time approximation. Assume that the duration of each time slot is 1 millisecond. Don't forget to indicate the transition probabilities on the arrows.



2. Find the long-term blocking probability **from the Markov chain**.

$$\begin{aligned} \lambda \delta p_0 &= \mu \delta p_1 & \lambda \delta p_1 &= 2 \mu \delta p_2 & \lambda \delta p_2 &= 3 \mu \delta p_3 \\ p_1 &= A p_0 & p_2 &= \frac{1}{2} A p_1 & p_3 &= \frac{1}{3} A p_2 \\ & & &= \frac{1}{2} A^2 p_0 & &= \frac{1}{6} A^3 p_0 \end{aligned}$$

$$p_0 + p_1 + p_2 + p_3 = 1 \Rightarrow p_0 = \frac{1}{1 + A + \frac{A^2}{2} + \frac{A^3}{6}} = \frac{1}{1 + 2 + 2 + \frac{8}{6}} = \frac{1}{5 + \frac{4}{3}} = \frac{3}{15 + 4} = \frac{3}{19}$$

3. **Use Erlang B formula**, find the blocking probability.

$$1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} = 1 + 2 + \frac{4}{2} + \frac{8}{6} = 5 + \frac{4}{3} = \frac{15 + 4}{3} = \frac{19}{3}$$

$$\text{Blocking Probability} = \frac{\frac{A^3}{3!}}{1 + A + \frac{A^2}{2!} + \frac{A^3}{3!}} = \frac{\frac{8}{6}}{\frac{19}{3}} = \frac{4}{19}$$